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Transverse Polarization of Top Quarks Produced at a Photon-Photon Collider

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Abstract:

At future $\gamma\gamma$ colliders copious production of $t\bar{t}$ pairs is possible. This would allow a detailed investigation of the interactions involving the top quark. We propose some orrelations which are sensitive to $t\bar{t}$ final state interactions and we compute the QCD and standard model Higgs boson contributions to these correlations. QCD induced transverse polarization of top quarks is found to be sizeable and measurable at a high-energy e^+e^- collider with an integrated luminosity of 10 (fb)$^{-1}$ which is converted into a photon collider by backscattering of laser photons.

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One of the attractive possibilities of a future high energy linear e^+e^- collider [1] is to convert it via backscattering of laser photons off the initial lepton beams into a high energy $\gamma\gamma$ collider [2]. It is expected that such a facility would provide a tool for a number of novel precision studies of strong and electroweak interactions [1, 3 - 8]. For instance copious production of top quarks is feasible. This would allow for some detailed studies of top quark physics [5 - 8] which would be to some extent complementary * to studies of $e^+e^- \to t\bar{t}$. One important aspect of top quark physics is the quasi-free behaviour of the top due to its heavy mass ($m_t > 130$GeV [11]). On average the top will have decayed before being able to form hadrons. This property makes the the spin polarization of the top a good observable as it can be traced through the angular distributions of the t and/or $\bar{t}$ decay products.

In this note we exploit this property and investigate the transverse polarization (i.e., the polarization transverse to the production plane) of the t and/or $\bar{t}$ produced in $\gamma\gamma$ collisions. This polarization which is due to $t\bar{t}$ final state interactions may serve as a probe of (non) standard model (SM) interactions [12 - 15] in the $t\bar{t}$ system. We propose observables by which this tranverse polarization can be traced in the t and $\bar{t}$ decay products and compute the dominant SM contributions to their expectation values.

We consider $t\bar{t}$ production via photon fusion:

$$\gamma(p_1) + \gamma(p_2) \to t(k_1, s_1) + \bar{t}(k_2, s_2) \tag{1}$$

where the momenta are defined in the photon-photon c. m. frame and s_1 and s_2 label the spins of t and $\bar{t}$. In the following we consider only unpolarized photon beams. The process may then be described by the density matrix:

$$R_{\alpha\alpha',\beta\beta'} = \sum{}' < t(k_1,\alpha')\bar{t}(k_2,\beta')|T|\gamma\gamma >^* < t(k_1,\alpha)\bar{t}(k_2,\beta)|T|\gamma\gamma > \tag{2}$$

where the $\sum'$ denotes averaging over the $\gamma\gamma$ polarizations. Note that R is an even function of the three-momentum $\mathbf{p_1}$ due to Bose symmetry of the two-photon state. With respect

* For $t\bar{t}$ production by beamstrahlung photons at a linear e^+e^- collider see [9,10].

to the $t\bar{t}$ spin space it has the following matrix structure:

$$R = AI \times I + \mathbf{B} \cdot \sigma \times I + \mathbf{C} \cdot I \times \sigma + D_{ij}\sigma^i \times \sigma^j \tag{3}$$

where σ^i denote the Pauli matrices and the first (second) factor in the tensor products refers to the $t(\bar{t})$ spin space. Due to rotation invariance the structure functions $\mathbf{B}$, $\mathbf{C}$ and D_{ij} can be decomposed in terms of the unit vectors $\hat{\mathbf{p}}_\mathbf{1}$, $\hat{\mathbf{k}}_\mathbf{1}$ and $\hat{\mathbf{n}} = \hat{\mathbf{p}}_\mathbf{1} \times \hat{\mathbf{k}}_\mathbf{1}/|\hat{\mathbf{p}}_\mathbf{1} \times \hat{\mathbf{k}}_\mathbf{1}|$ which is orthogonal to the production plane. For $\mathbf{B}$ and $\mathbf{C}$ one can write:

$$\mathbf{B} = b_1\hat{\mathbf{p}}_\mathbf{1} + b_2\hat{\mathbf{k}}_\mathbf{1} + b_3\hat{\mathbf{n}}, \quad \mathbf{C} = c_1\hat{\mathbf{p}}_\mathbf{1} + c_2\hat{\mathbf{k}}_\mathbf{1} + c_3\hat{\mathbf{n}}, \tag{4}$$

while D_{ij} which characterizes the correlation between the t and $\bar{t}$ spins can be decomposed in terms of 8 independent scalar functions. For the tree-level SM amplitude one has $\mathbf{B} = \mathbf{C} = \mathbf{0}$, i.e., the top quarks are not polarized. If time reversal (T) invariance holds the structure functions b_3 and c_3, which lead to t and $\bar{t}$ polarization transverse to the production plane, become nonzero only due to the interference between the dispersive part and the absorptive part of the scattering amplitude for the process (1). It is easy to show that due to Bose symmetry of the $\gamma\gamma$ state the relations

$$b_3(x) = -b_3(-x), \quad c_3(x) = -c_3(-x) \tag{5}$$

must hold, where $x = \hat{\mathbf{p}}_\mathbf{1} \cdot \hat{\mathbf{k}}_\mathbf{1}$ is the cosine of the scattering angle. This implies that at the level of $t\bar{t}$ final states the spin projection $\mathbf{s_1} \cdot \hat{\mathbf{n}}$, where $\mathbf{s_1} = \frac{1}{2}\sigma \times I$, must be weighted with x in order to have a non-zero expectation value, and likewise for the corresponding $\bar{t}$ spin projection. Furthermore, one may wish to disentangle final-state interaction effects from CP-violating phenomena, which may also occur in the $t\bar{t}$ system. For detecting the former effects it is useful to employ T-odd (i.e., odd under reflection of momenta and spins) but CP-even correlations [14]. Hence for studying final state interactions in the process (1) which are induced by CP-invariant interactions the appropriate observable would be

$$x(\mathbf{s_1} + \mathbf{s_2}) \cdot \hat{\mathbf{n}} = \frac{x}{2}(\sigma \times I + I \times \sigma) \cdot \hat{\mathbf{n}} \tag{6}$$

Of course this cannot be used in an experiment because measurements of the t and $\bar{t}$ spin cannot be made on an event-by-event basis.

As mentioned above the top quark analyzes its spin by its parity-violating weak decay $t \to W + b$. It is well-known that the charged lepton from subsequent W decay is an efficient spin analyzer of the t quark [16]. In order to construct realistic observables we consider first the case where both t and $\bar{t}$ decay semileptonically, i.e.,

$$\begin{aligned} t &\to W^+ + b \to \ell^+ + \nu_\ell + b \\ \bar{t} &\to W^- + \bar{b} \to \ell^- + \bar{\nu}_\ell + \bar{b} \end{aligned} \tag{7}$$

A CP-even but T-odd observable can then be formed using the momenta $\mathbf{q}_\pm$ of the charged leptons $\ell^\pm$ measured in the e^+e^- laboratory frame and the direction $\hat{\mathbf{p}}$ of the electron beam:

$$O_L = \frac{1}{m_t^3}\{\hat{\mathbf{p}} \cdot (\mathbf{q}_+ + \mathbf{q}_-)\}\hat{\mathbf{p}} \cdot (\mathbf{q}_+ \times \mathbf{q}_-) \tag{8}$$

where the top mass m_t is used to make O_L dimensionless.

The other useful type of events results from semileptonic t and non-leptonic $\bar{t}$ decay and vice versa. (These will be called "semihadronic" below.) For these events the momentum direction $\hat{\mathbf{k}}_-$ ($\hat{\mathbf{k}}_+$) of the $\bar{t}(t)$ can be reconstructed. (Again we refer to the laboratory frame.) For the respective channels we can then use the T-odd observables:

$$\begin{aligned} O_{B_1} &= \frac{1}{m_t}(\hat{\mathbf{p}} \cdot \hat{\mathbf{k}}_-)\hat{\mathbf{p}} \cdot (\hat{\mathbf{k}}_- \times \mathbf{q}_+) \\ O_{B_2} &= \frac{1}{m_t}(\hat{\mathbf{p}} \cdot \hat{\mathbf{k}}_+)\hat{\mathbf{p}} \cdot (\hat{\mathbf{k}}_+ \times \mathbf{q}_-). \end{aligned} \tag{9}$$

As these observables are intimately related to the t and $\bar{t}$ spin-momentum projections $x\mathbf{s_{1,2}}\cdot\hat{\mathbf{n}}$, they are more sensitive to $t\bar{t}$ final-state interaction effects than (8). The translation of the spin-momentum correlation (6) into a final state observable is $O_{B_1} - O_{B_2}$ which is CP-even. Below we shall evaluate:

$$B_t = < O_{B_1} > - < O_{B_2} > . \tag{10}$$

In this quantity contributions from CP-invariant absorptive parts add up.

Within the SM the dominant contributions to the absorptive part of the scattering amplitude of (1) arise from QCD corrections and possibly also from Higgs boson (H) exchange due to the sizeable Yukawa coupling of the top quark. Of the QCD corrections, only gluon exchange between the final t and $\bar{t}$ quarks leads to an absorptive part of the one-loop amplitude. An absorptive part from Higgs boson interactions arises from H exchange between the final t and $\bar{t}$ and from s-channel H exchange with a $\gamma\gamma H$ vertex being induced by W boson and t quark loops. However, interference of the s-channel diagrams with the Born amplitude does not induce a transverse polarization of the top quark.

In general, the interference between the Born amplitude and the absorptive part will give contributions not only to b_3 and c_3 but also to some terms in D_{ij}. However, in the case at hand, only b_3 and c_3 become nonzero. Gluon exchange gives:

$$\begin{aligned} b_3^{gluon} = c_3^{gluon} &= \alpha_s e^4 Q_t^4 |\hat{\mathbf{p}}_\mathbf{1} \times \hat{\mathbf{k}}_\mathbf{1}| (m_t/\beta E_1) \cdot (1-\beta^2 x^2)^{-1} \cdot (x^2-1)^{-1} \\ &\cdot [4\beta x(1-x^2) + 2\beta x(-5\beta^2 - 6\beta - 1)\ln(\beta+\beta^2) \\ &+ 2\beta x(5\beta^2 - 6\beta + 1)\ln(\beta - \beta^2) \\ &+ 2\beta(-2\beta^2 x^2 - 3\beta^2 + 6\beta x - x^2)\ln(\beta - \beta^2 x) \\ &+ 2\beta(2\beta^2 x^2 + 3\beta^2 + 6\beta x + x^2)\ln(\beta + \beta^2 x)] \end{aligned} \tag{11}$$

where Q_t is the charge of the top quark in units of e, E_1 is the photon energy, $\beta = (1 - m_t^2/E_1^2)^{1/2}$ is the velocity of the top quark and x is the cosine of the scattering angle defined above. Higgs boson exchange induces:

$$\begin{aligned} b_3^{Higgs} &= c_3^{Higgs} = \frac{3e^4 Q_t^4}{32\pi}\frac{m_t^2}{v^2} |\hat{\mathbf{p}}_\mathbf{1} \times \hat{\mathbf{k}}_\mathbf{1}| (m_t/E_1)(1-\beta^2 x^2)^{-1}(x^2-1)^{-1} \\ &\cdot \{2\beta^2 x(x^2 + 2d_H - 3)\ln\frac{1+\beta}{1-\beta} + 2\beta x(x^2 d_H - 3d_H + 2)\ln\frac{d_H+1}{d_H-1} \\ &+ \beta x(-\beta^2 x^2 d_H - \beta^2 x^2 - 2\beta^2 d_H^2 + 3\beta^2 d_H + \beta^2 + 2d_H - 3)(F(\beta) + F(-\beta)) \\ &+ (\beta^2 x^4 + 3\beta^2 x^2 d_H - 2\beta^2 x^2 - 2\beta^2 d_H^2 + \beta^2 d_H - \beta^2 - x^2 + 1)(F(\beta) - F(-\beta)) \\ &+ 4\beta x(1-x^2)\}, \quad d_H = 1 + \frac{M_H^2}{2E_1^2\beta^2}, \\ F(\beta) &= \frac{1}{R}\ln\frac{d_H - \beta x + R}{d_H - \beta x - R}, \quad R = \sqrt{(1-\beta x d_H)^2 + \beta^2(d_H^2 - 1)(1-x^2)} \end{aligned} \tag{12}$$

where v =246 GeV. Using in addition the contribution of the Born ampitude to R, which is easily calculated, we can now compute the expectation values of (8) and (9). As further ingredients for this calculation one needs the distributions of polarized t and $\bar{t}$ decay into the respective channels. We take the distributions as obtained from the SM Born decay amplitudes. Finally one requires the $\gamma\gamma$ luminosity spectrum which we take from [2]. We assume the energy of the laser photon to be 1.26eV and the $e-\gamma$ conversion factor is taken to be one. Then, choosing two different values for the top mass, we obtain for a e^+e^- collider at center-of-mass energy $\sqrt{s} = 500\text{GeV}$:

$$\begin{aligned} &< O_L > = 0.0037\alpha_s, \quad B_t = 0.11\alpha_s, \quad \text{for } m_t = 130\text{GeV}, \\ &< O_L > = 3.9 \times 10^{-4}\alpha_s, \quad B_t = 0.047\alpha_s, \quad \text{for } m_t = 170\text{GeV}. \end{aligned} \tag{13}$$

For $\sqrt{s} = 1\text{TeV}$ we get:

$$\begin{aligned} &< O_L > = 0.041\alpha_s, \quad B_t = 0.23\alpha_s, \quad \text{for } m_t = 130\text{GeV}, \\ &< O_L > = 0.016\alpha_s, \quad B_t = 0.18\alpha_s, \quad \text{for } m_t = 170\text{GeV}. \end{aligned} \tag{14}$$

Eqs. (13) and (14) do not contain the contributions from Higgs boson exchange. They are very small and of the order of 10^{-5} to 10^{-8}. In order to estimate the sensitivity of the correlations to the QCD induced transverse polarization we need the effective $t\bar{t}$ cross sections (i.e., the cross sections for the process (1) folded with the $\gamma\gamma$ luminosity spectrum) and the width of the distribution of the observables $\Delta O = (< O^2 > - < O >^2)^{1/2} \simeq (< O^2 >)^{1/2}$. For $m_t = 130\text{GeV}$:

$$\begin{aligned} &\sigma_{t\bar{t}} = 0.52\text{pb}, \quad < O_L^2 >= 0.0016, \quad < O_{B_1}^2 >= 0.0065, \ \text{at } \sqrt{s} = 500\text{GeV}, \\ &\sigma_{t\bar{t}} = 0.995\text{pb}, \quad < O_L^2 >= 0.038 \quad < O_{B_1}^2 >= 0.005, \ \text{at } \sqrt{s} = 1000\text{GeV}. \end{aligned} \tag{15}$$

and for $m_t = 170\text{GeV}$:

$$\begin{aligned} &\sigma_{t\bar{t}} = 0.09\text{pb}, \quad < O_L^2 >= 0.00026 \quad < O_{B_1}^2 >= 0.0056, \ \text{at } \sqrt{s} = 500\text{GeV}, \\ &\sigma_{t\bar{t}} = 0.60\text{pb}, \quad < O_L^2 >= 0.0092 \quad < O_{B_1}^2 >= 0.0044, \ \text{at } \sqrt{s} = 1000\text{GeV}. \end{aligned} \tag{16}$$

The numbers in eqs. (13),(14) and (15),(16) show that the sensitivity – signified by the signal-to-noise ratios $< O > /\Delta O$ – of the observables (9) is much higher than this of

the one obtainable with (8). For $m_t = 130$ GeV we have $B_t/(\sqrt{2}\Delta O_{B_1}) \simeq 0.10(0.23)$ at $\sqrt{s} = 500(1000)$ GeV. In order to establish this correlation as a, say, 4σ effect about 1600 (320) "semihadronic" $t\bar{t}$ events are required at the respective energies, which is feasible with an luminosity of 10 $(\text{fb})^{-1}$ for e^+e^- colliders. For $m_t = 170$ GeV the corresponding numbers are $B_t/(\sqrt{2}\Delta O_{B_1}) \simeq 0.044(0.2)$ at $\sqrt{s} = 500(1000)$ GeV. Using the cross sections of (16) one sees that in this case one obtains only a 1σ effect at 500 GeV, whereas the correlation (9) would be clearly measurable (as a 10σ effect with the above luminosity) at 1 TeV.

In conclusion: we have shown that the triple product correlations (9) are sensitive tools for detecting transverse polarization of t and $\bar{t}$ produced in $\gamma\gamma$ collisions. Transverse polarization of the top quark is due to radiative corrections and results within the SM primarily from QCD final state interactions. As the couplings of the top quark have yet to be measured in future experiments the observables above may be used for a detailed investigation of the forces in the $t\bar{t}$ system.

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